Homework 6

Problem 2: Cosmological Distance Ladder

The Cosmological Distance Ladder is what is used to measure distances to extremely distant objects by using a series of measurements ("rungs") that depend on the measurement before it.

Rung #1: Size of the Earth

The Greek Eratosthenes first used the shadow cast by the sun at two different points to geometrically determine the size of the earth. The present day value of the Earth's radius is $R_\oplus \approx 6400$ km.

Rung #2: Distance to the Moon

The Greek Aristarchus used the shadow of the Earth during lunar eclipses (where the moon passes through the Earth's shadow) to geometrically determine the distance to the moon in terms of the Earth's radius. The present day value of this distance is $D_{moon} \approx 60 R_\oplus \approx 384,000$ km.

Rung #3: Distance to the Sun

The Greek Aristarchus used the timing differences between the moon's 1st and 3rd quarter phases to geometrically determine the distance to the sun in terms of the Earth's radius. The present day value of this distance is $D_\odot \approx 390 D_{moon} \approx 390 \times 60 R_\oplus \approx 1.5 \times 10^8$ km. The Earth-Sun distance is defined to be 1 Astronomical Unit: $D_{sun} \equiv 1$ AU.

Rung #4: Distance to nearby stars and clusters

This method uses parallax, the apparent change in position of stars as the Earth orbits around the Sun. The distance D (in parsecs) to a given star or cluster is given in terms of its parallax angle α (in arcseconds): $D = 1/\alpha$. A parsec ("parallax arcsecond", pc for short) is approximately equal to 206,000 AU $\approx 206,000 \times 390 \times 60 R_\oplus \approx 4.82 \times 10^9 R_\oplus$. This method will measure distance out to a few hundred pc.

Rung #5: Main Sequence fitting

Sometimes called spectroscopic parallax, this method exploits the colors of stars to determine their distance. One can plot the colors of stars (of a single cluster) against their (apparent) magnitude to form the Hertzsprung-Russel (HR) diagram. If one then plots a second cluster of stars (at a different distance), the main sequence of stars of the second will be displaced (in magnitude) from the first. The amount of displacement is related to their relative distances:

$$m_1 - m_2 = -2.5 \log\left(\frac{I_1}{I_2}\right) = -5 \log\left(\frac{D_2}{D_1}\right)$$

where m, I, D are the apparent magnitudes, intensities and distances to the two clusters. Knowing the difference in magnitudes, the distance ratio can be computed. This method will measure distances out to a few kiloparsecs (a few kpc).

Rung #6: RR Lyrae

RR Lyrae stars are a specific type of variable star that is used as a "s-tandard candle" (meaning, we presume to know their intrinsic luminosity). Knowing the (apparent) magnitude (m_1) of an RR Lyrae star in a (globular) cluster, whose distance (D_1) is known (from the previous rungs), the distance D_2 to a different RR Lyrae star (of magnitude m_2) is given by:

$m_1 - m_2 = -5 \log \left(\frac{D_2}{D_1}\right)$

This method can be used to measure distances out to approximately 10 kpc.

Rung #7: Cepheids

Cepheids are a class of variable stars (found in open and globular clusters) also used as a standard candle. Cepheids have an absolute magnitude that is related to their period of variability (specifically, log(m) $\propto$ log(P)). Knowing the period and magnitude m_2 of a Cepheid at an unknown distance D_2, one can use the period-magnitude relation to compare to a Cepheid of the same period (with known magnitude m_1 and distance D_1):

$m_1 - m_2 = -5 \log \left(\frac{D_2}{D_1}\right)$

This method can be used to measure distances out to approximately ten million parsecs (10 Mpc).

Rung #8: Hubble's Law

Using Cepheids, one can measure distances to other galaxies. Hubble's discovery that the recessional velocity of a galaxy (measured by its redshift) is linearly related to the distance allows one to determine the distance to further galaxies: $V = H_0 D$, where V is the recessional velocity, D is the distance, and $H_0 = 100h$ km s^{-1} Mpc^{-1}.

This method can be used to measure distances out to approximately 4 Gpc, where a Gpc $= 10^9$ pc $\approx 10^9 \times 206,000$ AU $\approx 10^9 \times 206,000 \times 390 \times 60 R_\oplus \approx 4.82 \times 10^{18} R_\oplus \approx 3 \times 10^{22}$ km.

Problem 3.

a) Assistant needs to measure the **apparent** luminosity (intensity or apparent magnitude) and colour of all the stars in the cluster. Colour can be measured by using different coloured filtres and comparing two of them (usually done with blue (B) and visual (V)). Temperature can be inferred from this colour difference.

(*note:* we can plot the **intrinsic** luminosity ONLY after the distance has been determined.)

(Another possibility is to take spectra (this takes a longer time) of the stars and classify them (O B A F G K M). Temperature can be inferred from the spectral type.)

b) The assistant needs to measure all the stars to establish a good HR diagram. Measuring only one star will not do. We now assume that properties of globular clusters are similar and thus the overall trend of the HR diagram should be the same for all globular clusters. We compare HR diagram of Rocky-II with an HR diagram of a cluster whose distance is known, say Kolb-0. Plot **apparent** luminosity (I) in the y-axis. Rocky-II and Kolb-0 have the same **intrinsic** (L) luminosity but due to their distance (d), their apparent luminosity does not appear to be the same.

$$
\begin{aligned}
I_{Rocky} &= \frac{L}{4\,\pi\,d_{Rocky}^{2}} \\
I_{Kolb} &= \frac{L}{4\,\pi\,d_{Kolb}^{2}}
\end{aligned}
$$

Since L is the same, we can calculate the distance to Rocky-II;

$$
I_{Rocky}\,d_{Rocky}^{2} = I_{Kolb}\,d_{Kolb}^{2} \implies d_{Rocky} = \sqrt{\frac{I_{Kolb}}{I_{Rocky}}}\,d_{Kolb}
$$

Problem 4.

As with all distance ladder problems, we will attempt to determine the distance to an object by comparing it to a similar, less-distant object. In this case, we observe a Cephied in galaxy M137 with a period of 10 days. It is well known that there is a strong relationship between period and luminosity for Cepheid stars. Thus, if we observe another Cepheid with a period of 10 days, we can assume it has the same luminosity as the Cephied in M137. We are given a graph with the period-apparent magnitude relationship for Cepheids in the Large Magellenic Cloud (LMC). From this graph, we can see that a Cepheid in the LMC with a period of 10 days (remember, $log(10days) = 1$) has an apparent magnitude of 15. We are also told that the distance to the LMC is 50kpc. We now have all the information needed to solve this problem. First, we use the equation for apparent magnitude:

$$m_1 - m2 = -2.5log(\frac{I_1}{I_2}) \tag{1}$$

Plugging in the appropriate numbers (with M137 and the LMC as objects 1 and 2, respectively, in the equation above) gives:

$$20 - 15 = -2.5log(\frac{I_{M137}}{I_{LMC}}) \tag{2}$$

Subtracting and dividing both sides by -2.5 yields:

$$-2 = log(\frac{I_{M137}}{I_{LMC}}) \tag{3}$$

Removing the logarithm by making both sides a power of 10 leaves us with:

$$10^{-2} = \frac{I_{M137}}{I_{LMC}} \tag{4}$$

Next, we use the equation for intensity:

$$I = \frac{L}{4\pi d^2} \tag{5}$$

and plug this into our results from above:

$$10^{-2} = \frac{\frac{L_{M137}}{4\pi d^2_{M137}}}{\frac{L_{LMC}}{4\pi d^2_{LMC}}} \tag{6}$$

Clearly, the 4π's in the previous equation cancel. Additionally, we are assuming that, since the Cepheids in the LMC and M137 have same period, they also have the same luminosity. That means they cancel as well, leaving only

$$10^{-2} = \frac{d^2_{LMC}}{d^2_{M137}} \tag{7}$$

Isolating d_{M137} on one side of the equation (since that's what we're trying to determine), we get:

$$d^2_{M137} = 10^2 d^2_{LMC} \tag{8}$$

We know that the distance to the LMC is 500kpc, so we can plug that number in above and solve to get $d_{M137} = 500kpc$.